



*Series: 7G Wireless  
Networks*

*gc*

*Report: #090819822 Quantum  
Computing with Continuous Variable*

*series: 7G Wireless Networks*

*report: #090819822 **QUANTUM COMPUTING  
WITH CONTINUOUS VARIABLE***

*by Savo Glisic  
Worcester Polytechnic Institute, Massachusetts  
GC Institute for Networking Sciences*

## *Preface*

*As a part of the solution to the constant demand for higher data rates, wireless communications are moving towards higher and higher frequencies including mmWave and THz bands. At the same time quantum physics is experimenting with quantum state transmission over sub-optical, THz and even much lower bands. In the anticipation of the development of quantum computer networks and quantum key distribution QKD over wireless networks, there is a need for design tools that will enable optimization of the heterogeneous networks that will seamlessly merge these two technologies as much as possible. At the same time, these networks will rely more and more on artificial intelligence so that further research is needed to integrate classical and quantum machine learning algorithms.*

*In general quantum technology can use either discrete (dv) or continuous (cv) variable information processing, where variables are modeled in the space of finite or infinite dimensions respectively. The latter option is what we cover in this report. While the former option, used in our recent book, is used for systematic introduction to the field of quantum computing the latter is more feasible for practical implementation and for this reason is in the focus of this report.*

*Here, we make an effort to provide a summary of an impressive work done so far by the quantum physics, computer science and artificial intelligence researchers and elaborate why and how it should serve as a basis for coming up with the solutions for integrated heterogeneous networks as defined above. We believe that 7G wireless networks will be based on this concept although the step-by-step application of this technology is already being proposed for 5G and will be seen in 6G as well.*

*When it comes to using the book for undergraduate and postgraduate courses we incorporate a number of DESIGN EXAMPLES to replace the classical concept of using “problems and solutions” addendums at the end of the chapters/book. This enables using more sophisticated assignments for the teamwork of the students. Our students have shown great enthusiasm for such approach.*

*In addition to universities the professionals in research, industry and regulatory institutions should benefit from the comprehensive coverage of the report.*

*Author  
March 2022*

*Amherst, Massachusetts*

<https://www.youtube.com/watch?v=u1XXjWr5frE>

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## *Ch1 INTRODUCTION*

### *1.1 Structure of the book*

Given the above objective, we present the overall material of the book within four chapters and in what follows we briefly summarize the content of these chapters.

*Ch 2 QUANTUM COMPUTING WITH CONTINUOUS VARIABLE:* The first part of the chapter introduces basic tools for the analysis of the continuous variable quantum systems, mainly: Position and momentum space, Momentum Operator, Translation Operator in Quantum Mechanics, Wave Function, Hamiltonian Operator, Schrödinger equation and Relativistic wave equations. In the second part of the chapter we use these tools to model and analyze a number of physical quantum channels including: Gaussian Quantum Information, Elements of Gaussian Quantum Information Theory, Distinguishability of Gaussian States, Measures of distinguishability, Distinguishing optical coherent states, Examples of Gaussian Quantum Protocols, Quantum teleportation and variants, Quantum cloning, Bosonic Gaussian Channels, One-mode Gaussian channels, Classical capacity of Gaussian channels, Quantum capacity of Gaussian channels, Quantum dense coding and entanglement-assisted classical capacity, Entanglement distribution and secret-key capacities, Gaussian channel discrimination and applications.

*Ch 3 ENTANGLEMENT:* Quantum entanglement occurs when two systems share a common quantum mechanical state. Such systems also share a common fate, even if they become physically quite separated. The results of a measurement performed on one will determine the results of future measurements on the second. This phenomenon is exploited in quantum key management in quantum cryptography enabling near absolute security. The first part of the chapter introduces parameters of cv channel relevant for entanglement including: Quantum information with continuous variables, Continuous Variables in Quantum Optics, The quadratures of the quantized field, Phase-space distribution, Gaussian states, Linear optics, Nonlinear optics, Polarization and spin and Phase reference.

The second section covers entanglement itself mainly: Continuous-Variable Entanglement, Bipartite entanglement, Multipartite entanglement, Nonlocality and Remote Entanglement Distribution. The chapter also includes a number of appendices providing additional explicit explanations for topics like: Schmidt decomposition, Mermin-Klyshko inequalities, The classification of tripartite three-mode Gaussian states

*Ch 4 ACHIEVABLE TRANSMISSION RATES:* Here we review the main results in the analysis of the achievable rates in quantum and related channels. In the first part of the chapter we base our presentation on the original works in the field and in the second part we revisit the topic with the fresh view based on the latest publications on the subject. The chapter starts with discussion of the Bosonic Gaussian channels that play a fundamental role in characterizing the efficiency of a variety of tasks in continuous-variables quantum information processing including quantum communication and cryptography. Most importantly, communication channels such as optical fibers can, to a good approximation, be described by Gaussian quantum channels. For that reason, although we have already invested a great deal of space to these channels so far, here we go further in depth with specific models of these channels and alternative presentations. Special care is taken to enlighten the physical interpretation of the boson particle itself. In the sequel the chapter presents discussion on Entanglement-Assisted Classical Capacity of Noisy Quantum Channels, Entanglement-assisted classical capacity, Entanglement-assisted capacity of quantum channel with additive constraint, Entanglement in Quantum channels with cv and Fundamental Limits of Quantum Communications.





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