



*Series: 7G Wireless
Networks*

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report: #090819823 Quantum Networks

series: 7G Wireless Networks

report:090819823 QUANTUM NETWORKS

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Preface

As a part of the solution to the constant demand for higher data rates, wireless communications are moving towards higher and higher frequencies including mmWave and THz bands. At the same time quantum physics is experimenting with quantum state transmission over sub-optical, THz and even much lower bands. In the anticipation of the development of quantum computer networks and quantum key distribution QKD over wireless networks, there is a need for design tools that will enable optimization of the heterogeneous networks that will seamlessly merge these two technologies as much as possible.

This is the focus of this report.

At the same time, these networks will rely more and more on artificial intelligence so that further research is needed to integrate classical and quantum machine learning algorithms.

In general quantum technology can use either discrete (dv) or continuous (cv) variable information processing, where variables are modeled in the space of finite or infinite dimensions respectively. While the former option, used in our recent book, is used for systematic introduction to the field of quantum computing the latter is more feasible for practical implementations and for this reason is in the focus of this series of reports.

Here, we make an effort to provide a summary of an impressive work done so far by the quantum physics, computer science and artificial intelligence researchers and elaborate why and how it should serve as a basis for coming up with the solutions for integrated heterogeneous networks as defined above. We believe that 7G wireless networks will be based on this concept although the step-by-step application of this technology is already being proposed for 5G and will be seen in 6G as well.

When it comes to using the book for undergraduate and postgraduate courses we incorporate a number of DESIGN EXAMPLES to replace the classical concept of using “problems and solutions” addendums at the end of the chapters/book. This enables using more sophisticated assignments for the teamwork of the students. Our students have shown great enthusiasm for such approach.

In addition to universities the professionals in research, industry and regulatory institutions should benefit from the comprehensive coverage of the book.

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<https://www.youtube.com/watch?v=uJXXjWr5frE>

Ch 1 INTRODUCTION

1.1 Structure of the book

Ch 2 QUANTUM NETWORK ROUTING

2.1 Routing over Virtual Quantum Network

2.1.1 Preliminaries

2.1.2 Ring Network

2.1.3 Sphere Network

2.1.3.1 Definition of the VQN architecture

2.1.3.2 Routing Algorithm

2.2 Minimum Cost Routing

2.2.1 Quantum Routing Parameters

2.2.2 Entanglement-Gradient Routing

2.3 Entanglement Distribution

2.3.1 Preliminaries

2.3.2 The optimal RED protocol

2.3.3 Stationary Protocol

2.4 Quantum graph

2.5 Multipoint entanglement distribution (multi-partite entanglement)

REFERENCES

Ch 3 Dynamic QUANTUM NETWORK TOPOLOGY DESIGN

3.1 Quantum graph states

3.1.1. Interaction pattern.

3.1.2. Stabilizer formalism.

3.1.3 Local Clifford group and LC equivalence.

3.1.4 Weyl operators and Heisenberg group

3.2. Evaluation of the Degree of Entanglement for Graph States

3.2.1 Schmidt measure

3.2.2. Generalization of the evaluation rules

3.3 Quantum State Graph Reconfiguration

3.3.1 Vertex-deletions and local complementations

3.3.2 Circle graphs

3.3.3 Examples of vertices

3.3.4 Graph Reconfiguration Algorithms

REFERENCES

Ch 4 QN STABILITY

4.1 Dynamic QN PROTOCOLS

4.1.1 Dynamic random subgraphs

4.1.2 Dynamic Quantum states

4.1.3 Quantum Decision Processes for QN Protocols

4.1.4 Performance Measures

4.1.5 Policy optimization

4.2 QN Stability

4.2.1 Stability of an entangled quantum network.

REFERENCES

Ch 5 SATELLITE QN

5.1 Elementary Link Generation with Satellites

5.2 Implementation Aspects of cv Satellite QN

5.2.1 Uplink/Downlink Free-Space Optical Channels

- 5.2.2 *CV Quantum Systems in Satellite Networks*
- 5.2.3 *Continuous Variable-QKD in Satellite Networks*
- 5.2.4 *Entanglement and CV-QK Distribution in Satellite Networks*
- 5.2.5 *Non-Gaussian CV Quantum Communication over Atmospheric Channels*

REFERENCES

Ch 6 Quantum Network Optimization

6.1 Algorithms

6.1.1 The Quantum Alternating Operator Ansatz (QAOA)

6.1.2 QAOA Mappings: Strings

6.1.2.1 Example: Max- κ -ColorableSubgraph

6.1.2.2 Full QAOA Mapping

6.1.3 QAOA Mappings: Orderings and Schedules

6.1.4 QAOA mappings for a variety of NP optimization problems

6.1.4.1 Bit-Flip (X) Mixers

6.1.4.2 Controlled-Bit-Flip ($\Lambda_f(X)$) Mixers

6.1.4.3 Mixers

6.1.4.4 Controlled-XY Mixers

6.1.4.5 Permutation Mixers

6.2 Multidomain Optimization of Quantum Network

6.2.1 Quantum-Domain Optimization (QDO)

6.2.2 Classical-Domain Optimization (CDO)

REFERENCES

Ch1 INTRODUCTION

1.1 Structure of the book

Given the above objective, we present the overall material of the book within six chapters and in what follows we briefly summarize the content of these chapters.

Ch 2 QUANTUM NETWORK ROUTING: A quantum network can enable long distance quantum communication and assemble small quantum devices into a large quantum computing cluster. Each network node can thereby be seen as a small, few qubit quantum computer. Qubits can be sent over direct physical links connecting nearby quantum nodes, or by means of teleportation over pre-established entanglement amongst distant network nodes. Such pre-shared entanglement effectively forms a shortcut - a virtual quantum link - which can be used only once.

Here, we present an abstraction of a quantum network, referred to as virtual quantum network VQN, that allows ideas from computer science to be applied to the problem of routing qubits and manage entanglement in the network. Specifically, we consider a scenario in which each quantum network node can create entanglement pairs with its immediate neighbors over a physical connection and perform entanglement swapping operations in order to create long distance virtual quantum links. Here, we discuss the features unique to quantum networks, which call for the development of new routing techniques. As an example, we present two simple hierarchical routing schemes for a quantum network of N nodes for a ring and sphere topology. For these topologies we present efficient routing algorithms requiring $O(\log N)$ qubits to be stored at each network node, $O(\text{polylog} N)$ time and space to perform routing decisions, and $O(\log N)$ timesteps to replenish the virtual quantum links in a model of entanglement generation. In addition the chapter covers discussion on *Minimum Cost Quantum Routing*, *Entanglement-Gradient Routing*, *Quantum graph and Multipoint entanglement distribution (multi-partite entanglement)*.

Ch 3 Dynamic QUANTUM NETWORK TOPOLOGY DESIGN: For further study of entanglement distribution, discussed in the previous chapter, especially when it comes to multiparty connections, we need more sophisticated tools like quantum graphs states theory in order to manage the network entanglement reconfiguration in dynamic networks.

We start with a detailed introduction of graph states, following up with study of their entanglement properties. After setting basic notations that are frequently used throughout this chapter, we give essentially two alternative definitions for graph states, namely, in terms of the underlying interaction pattern and in terms of the stabilizer. We illustrate how elementary properties of a graph state and basic operations, such as (Pauli) measurements, on these states, can be phrased concisely in terms of the underlying graph. It is shown that the action of Clifford operations on graph states and the reduced states for graph can be determined efficiently from the underlying graph. These relations will allow for a classification of graph states later and for an efficient computation of entanglement properties. In addition to multiparty quantum communication, we also briefly discuss some examples and applications to quantum error correction, and quantum computation. These problems will be then discussed in more detail in subsequent chapters. A short review about possible realizations of graph states in physical systems is also covered. Then we discuss the classification of graph states in terms of equivalence classes under different types of local operations and provide a complete classification for graph states with up to seven vertices. The results indicate that graph states form a *feasible and sufficiently rich class of multiparty entangled states*, which can serve as good starting point for studies of multi - party entanglement. We further review some results about the ‘non - classicality’ of graph states and how their entanglement can be detected with Bell inequalities. The genuine multi - particle entanglement of graph states is characterized and quantified in terms of the Schmidt measure, to which we provide upper and lower bounds in graph theoretical terms. *Graph Reconfiguration Algorithms used to manage the network’s dynamics are discussed in detail.*

Ch 4 QN STABILITY: Here, we present a mathematical model to quantify the dynamics of entangled network structures and entanglement flow in the quantum Internet. The analytical solutions of the model determine the equilibrium states of the entangled quantum networks and characterize the stability, fluctuation attributes, and dynamics of entanglement flow in entangled network structures.

To quantify the dynamic attributes of entangled structures of the quantum Internet, the analytical model defines a stability function motivated by the free energy thermodynamical potential function in thermodynamics and statistical physics. The concept of stability function has therefore essentially roots in the Le Chatelier principle in a chemical equilibrium. The Le Chatelier principle says that chemical equilibrium occurs at minimum Gibbs energy of the reactants and the products and disturbance of the mix would result in restoration of the equilibrium in a way that cancels the perturbation. In the

developed model, the stability function determines the equilibrium state of the entangled structure. A stable equilibrium state of the entangled quantum network N is stable if heavy fluctuations in the network have zero effect on the entanglement flow in the entangled quantum network. If stability function is in a global minimum, then the entangled structure is in a stable equilibrium state. The determination of the stable equilibrium states of an entangled structure is fundamental to any seamless communication in a global-scale quantum Internet. The seamless quantum communication refers to a stable (reliable) transmission without fluctuations (the fluctuation does not exceed a critical limit). In a stable network state, the probability of non-erroneous information transmission between nodes and at moment is above a critical bound C^* , $R_{xy}(t_0, t) > C^*$, given that at moment t_0 the communication is correct. The entanglement flow is considered seamless optimal if it is seamless and if the entanglement rate exceeds a critical lower bound set for the entangled connections. Here, we quantify the stability function for various entangled structures. The reliability of quantum communication is analyzed via the stability function of the entangled quantum network since the stability of the entangled structure implies the reliability of quantum communication within the network.

Ch 5 SATELLITE QN: Here we consider the model of elementary link generation, in which the entanglement sources are placed on satellites orbiting the earth. Satellites are one of the best methods for achieving global-scale quantum communication with current and near-term resources. Several proposals for satellite-based quantum networks have been made that use satellite-to-ground transmission, ground-to-satellite transmission, or both. Recent experiments between a handful of nodes have opened up the possibility of building a global-scale quantum internet using satellites. Within the chapter we discuss possible solutions and review existing results in the field.

Ch 6 QUANTUM NETWORK OPTIMIZATION: Nowadays, challenging computational problems arising in the practical world are frequently tackled by heuristic algorithms. These algorithms are empirically shown to be effective, but they have not been analytically proven to be the best approach, or even to outperform the best approach already published. Until recently, empirical investigation of quantum algorithms has been limited to tiny problems, given the typically exponential overhead of simulating quantum algorithms on classical processors. As prototype quantum hardware emerges which enables experimentation beyond what is reachable by even the world's largest supercomputers, we come into a new era for quantum heuristic algorithms.

A key question is: "What are good quantum heuristic algorithms to try?" A possible candidate is quantum approximate optimization algorithm, a quantum gate-model meta-heuristic which alternates between applying unitaries drawn from two families, a cost function based unitary family $U_P(\gamma) = e^{-i\gamma H_f}$ and a family of mixing unitaries $U_M(\beta) = e^{-i\beta H_B}$, for some fixed cost function based Hamiltonian H_f and some fixed mixing Hamiltonian H_B . The chapter presents a variety of the algorithms derived from this approach and compare their performance.



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