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*Series: 7G Wireless  
Networks*

*report: #310719493 QUANTUM  
Coding and CRIPTOGRAPHY*

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*report: #310719493 QUANTUM Coding and CRIPTOGRAPHY*

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*Preface: The research and practical results on Quantum computers in the recent years have given a major setback to classical and widely used cryptography schemes such as RSA (Rivest-Shamir-Adleman) Algorithm and ECC (Elliptic Curve Cryptography). RSA and ECC depend on integer factorization problem and discrete logarithm problem respectively, which can be easily solved by Quantum Computers of sufficiently large size running the infamous Shor's Algorithm. Therefore, cryptography schemes which are difficult to solve in both traditional as well as Quantum Computers need to be evaluated. **This report provides a detailed survey on Post-Quantum Cryptography schemes and emphasizes on their applicability to provide security in constrained devices.** A comprehensive insight is provided into the schemes which could possibly replace RSA and ECC for security in constrained devices.*

*While post-quantum cryptography is an effort to develop novel classical cryptosystems which are robust to factorization and other quantum algorithms, which is certainly one option, this does not completely solve the problem. The point is that there may be undiscovered quantum algorithms (or undiscovered classical ones) that might easily break the security of the new cryptosystems. In other words, postquantum cryptography is likely to offer only a partial and temporary solution to the problem. By contrast, quantum key distribution (QKD), discussed also in this series, offers the ultimate solution: restoring security and confidentiality by resorting to unbreakable principles of nature, such as the uncertainty principle or the monogamy of entanglement. **So we cover in details in this report the quantum cryptography as well.***

*Even though QKD offers the ultimate solution to the security problem, its ideal implementation is hard to implement in practice and there are a number of open problems to be addressed. On one side, fully-device independent QKD protocols provide the highest level of quantum security, but they are quite demanding to realize and are characterized by extremely low secret key rates. On the other hand, more practical QKD protocols assume some level of trust in their devices, an assumption that allows them to achieve reasonable rates, but this also opens the possibility of dangerous side-channel attacks.*

*Besides a trade-off between security and rate, there is also another important one which is between rate and distance. Today, we know that there is a fundamental limit which restricts any point to point implementation of QKD. Given a lossy link with transmissivity  $\eta$ , two parties cannot distribute more than the secret key capacity of the channel, which is  $-\log_2(1 - \eta)$ , i.e., a scaling of  $1.44\eta$  secret bits per channel use at long distance. Ideal implementations of QKD protocols based on continuous-variable systems and Gaussian states may approach this capacity, while those based on discrete variables falls below by additional factors. To overcome this limit and enable long-distance high-rate implementations of QKD, we need the develop quantum repeaters and quantum networks. In this way, we may achieve better long-distance scaling and further boost the rates by resorting to more complex routing strategies. The study of quantum repeaters and secure QKD networks is one of the hottest topics today which is also covered in this series. The series aims at providing an overview of the most important and most recent advances in the field of quantum cryptography, both theoretically and experimentally.*

*In near term, we expect that quantum security and QKD will be competing with so called post quantum security solutions and for this reason in a separate report of this series we discuss in details pros and cons of each technology.*

*When it comes to using the book for undergraduate and postgraduate courses we incorporate a number of DESIGN EXAMPLES in each report to replace the classical concept of using "problems and solutions" addendums at the end of the chapters/book. This enables using more sophisticated assignments for the teamwork of the students. Our students have shown great enthusiasm for such approach.*

*In addition to universities the professionals in research, industry and regulatory institutions should benefit from the comprehensive coverage of different report of the series.*

<https://www.youtube.com/watch?v=u1XXjWr5frE>

## *Ch 1 INTRODUCTION*

### *1.1 Structure of the book*

## *Ch 2 ELEMENTS of QUANTUM CODING THEORY*

### *2.1 Quantum coding theorems*

#### *2.1.1 Preliminaries*

#### *2.1.2 Quantum coding theorem*

##### *2.1.2.1 Channels with pure signal states*

#### *2.1.3 Reliability function*

#### *2.1.4 Reliability Function for Different Quantum Channel Examples*

### *2.2 Error Correction Limits for Quantum Metrology*

#### *2.2.1. Quantum Metrology in Presence of Impairments*

#### *2.2.2 Error Correction Enhanced Quantum Metrology*

##### *2.2.2.1. Noiseless Ancilla and Perfect Error Correction*

##### *2.2.2.2. Noisy Ancilla and Perfect Error Correction*

##### *2.2.2.3. Noiseless Ancilla and Imperfect Error Correction*

##### *2.2.2.4. Limitations of Current Quantum Technologies*

#### *2.2.3. Other Error Correction Strategies*

### *2.3 Stabilizer Codes*

#### *2.3.1 Stabilizer Coding*

### *2.4 Quantum LDPC Codes*

#### *2.4.1 An Introduction to classical LDPC Codes*

##### *2.4.1.1 Representations of LPDC Codes*

##### *2.4.1.2 LDPC Code Design Techniques*

##### *2.4.1.3 Iterative Decoding Algorithms*

#### *2.4.2 Constructing regular quantum LDPC codes*

### *2.5 Homological family of quantum LDPC codes*

#### *2.5.1 Code construction based on a Regular Tessellation of Hyperbolic Space*

#### *2.5.2 Hyperbolic 4-space and its Regular Tessellation by Hypercubes*

#### *2.5.3 Compact Manifolds*

#### *2.5.4 Code Performance*

#### *2.5.5 Decoders*

### *2.6 Quantum Inspired Space-Time Block Code*

#### *2.6.1 Emulated Quantum Channels*

#### *2.6.2 qSTBC Modelling*

#### *2.6.3 A Stabilizer Code for The Space-Time Channel*

#### *2.6.4 ML Decoder*

## *REFERENCES*

## *Ch 3 POST-QUANTUM CRYPTOGRAPHY*

### *3.1 Overview of Post-Quantum Cryptosystems*

#### *3.1.1 Multivariate cryptography*

#### *3.1.2 Lattice based cryptography*

#### *3.1.3 Hash based cryptography*

#### *3.1.4 Code based cryptography*



- 3.2 *Rainbow*
  - 3.2.1 *Multivariate Public Key Cryptography*
  - 3.2.2 *Rainbow Algorithm Specification*
  - 3.2.3 *Key Generation Speed Up*
  - 3.2.4 *Resistance to Attacks*
- 3.3 *NTRU N-th degree Truncated polynomial Ring Units*
  - 3.3.1 *Specification of NTRU Cryptosystem*
  - 3.3.2 *Security of NTRU*
- 3.4 *LWE Cryptosystem*
  - 3.4.1 *Preliminaries*
  - 3.4.2 *LWE Algorithm Variants*
  - 3.4.3 *LWE Public Key Cryptosystem*
- 3.5 *BLISS (Bimodal Lattice Signature Scheme (BLISS))*
  - 3.5.1 *BLISS: A Lattice Signature Scheme using Bimodal Gaussians*
  - 3.5.2 *Implementation of BLISS*
- 3.6 *Variants of Merkle Signature Scheme*
  - 3.6.1 *The Winternitz One-time Signature Scheme*
  - 3.6.2 *The Merkle Signature Scheme*
  - 3.6.3 *CMSS*
- 3.7 *Lamport Signature*
  - 3.7.1 *Improved Lamport one-time signature*
- 3.8 *McEllice Cryptosystem: Code-based cryptography*
  - 3.8.1 *McEliece Cryptosystem Using Extended Golay Code*
- 3.9 *Niederreiter Cryptosystem*
  - 3.9.1 *Niederreiter cryptosystems and Quasi-Cyclic Codes*
  - 3.9.2 *Subgroup  $K$  is indistinguishable*
  - 3.9.3 *Fault Attack on the Niederreiter Cryptosystem*
    - 3.9.3.1 *Binary Goppa Codes*
    - 3.9.3.2 *Binary Irreducible Goppa Cryptosystems*
    - 3.9.3.3 *The BIG-N Fault Injection Framework*
    - 3.9.3.4 *Constant and Quadratic Fault Injection Sequences*
    - 3.9.3.5 *The BIG-N Fault Attack*

*Appendix 3.1 Key Generation for a SIS-Based Scheme*

**REFERENCES**

## **Ch 4 QUANTUM CRYPTOGRAPHY**

- 4.1 *Discrete Variable Protocols*
  - 4.1.1 *Prepare and measure protocols*
  - 4.1.2 *Countermeasures*
  - 4.1.3 *Entanglement-based QKD*
  - 4.1.4 *Two-way quantum communication*
- 4.2 *Device-Independent QKD*
  - 4.2.1 *The link between Bell violation and unpredictability*
  - 4.2.2 *Performance bounds*
  - 4.2.3 *Protocols for DI-QKD*
  - 4.2.4 *Implementation of DI-QKD protocols*
- 4.3 *Continuous-Variable QKD*

- 4.3.1. *One-way CV-QKD protocols*
- 4.3.2. *Two-way CV-QKD protocols*
- 4.4 *Theoretical Models of Security*
  - 4.4.1 *Heisenberg's uncertainty principle*
- 4.5 *Limits of Point-to-Point QKD*
  - 4.5.1 *Adaptive protocols and two-way assisted capacities*
  - 4.5.2 *General weak-converse upper bound*
  - 4.5.3 *LOCC simulation of quantum channels*
  - 4.5.4 *Teleportation covariance and simulability*
  - 4.5.5 *Strong and uniform convergence*
  - 4.5.6 *Stretching of an adaptive protocol*
  - 4.5.7 *Upper bound simplification for two-way assisted capacities*
  - 4.5.8 *Bounds for teleportation-covariant channels*
  - 4.5.9 *Capacities for distillable channels*
- 4.6 *QKD Against a Bounded Quantum Memory*
  - 4.6.1 *Entropic uncertainty relations*
  - 4.6.2 *Bounded quantum storage model*
  - 4.6.3 *Quantum data locking*
- Appendix 4. A: Formulas for Gaussian states*

# Ch1 INTRODUCTION

## 1.1 Structure of the book

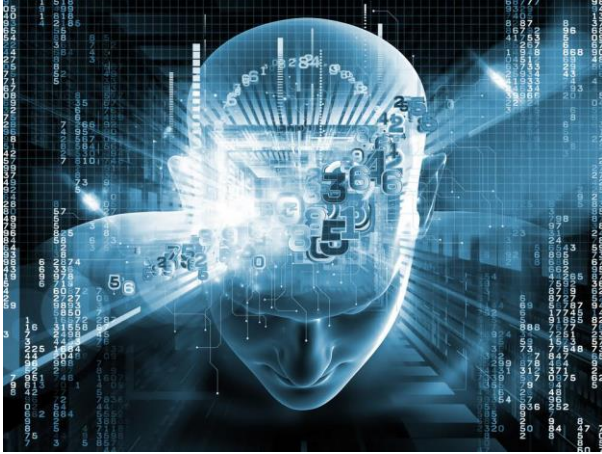
*Given the above objective, we present the overall material of the book within four chapters and in what follows we briefly summarize the content of these chapters.*

*Ch 2 ELEMENTS of QUANTUM CODING THEORY* When discussing the achievable rates in quantum channel, we were referring quite often to channel coding theorem. Before we turn to quantum channel and error correction coding, here we first summarize quantum coding theorems in more detail. This includes: This Quantum coding theorem, Reliability function, Reliability Function for Different Quantum Channel Examples, Error Correction Limits for Quantum Metrology, Quantum Metrology in Presence of Impairments, Error Correction Enhanced Quantum Metrology, Noiseless Ancilla and Perfect Error Correction, Noisy Ancilla and Perfect Error Correction, Noiseless Ancilla and Imperfect Error Correction, Limitations of Current Quantum Technologies, Other Error Correction Strategies, Stabilizer Codes and Stabilizer Coding.

*Ch 3 POST-QUANTUM CRYPTOGRAPHY* The research and practical results on Quantum computers in the recent years have given a major setback to classical and widely used cryptography schemes such as RSA (Rivest-Shamir-Adleman) Algorithm and ECC (Elliptic Curve Cryptography). RSA and ECC depends on integer factorization problem and discrete logarithm problem respectively, which can be easily solved by Quantum Computers of sufficiently large size running the infamous Shor's Algorithm. Therefore, cryptography schemes which are difficult to solve in both traditional as well as Quantum Computers need to be evaluated. This chapter provides a detailed survey on Post-Quantum Cryptography schemes and emphasizes on their applicability to provide security in constrained devices. A comprehensive insight is provided into the schemes which could possibly replace RSA and ECC for security in constrained devices.

*Ch 4 QUANTUM CRYPTOGRAPHY* This chapter aims at providing an overview of the most important and most recent advances in the field of quantum cryptography, both theoretically and experimentally. After a brief introduction of the general notions, we will review the main QKD protocols based on discrete- and continuous-variable systems. We will consider standard QKD, device-independent and measurement-device independent QKD. We will discuss the various levels of security for the main communication channel, from asymptotic security proofs to analyzes accounting for finite-size effects and composability aspects. We will also briefly review quantum hacking and side-channel attacks. Then, we will present the most recent progress in the exploration of the ultimate limits of QKD. In particular, we will discuss the secret key capacities associated with the most important models of quantum channels over which we may implement point to-point QKD protocols, and their extension to quantum repeaters and networks. Practical aspects of quantum repeaters will then be thoroughly discussed. Finally, we will treat topics beyond QKD, including quantum data locking, quantum random number generators, and quantum digital signatures.

Here we consider both discrete-variable systems, such as qubits or other quantum systems with finite-dimensional Hilbert space, and continuous-variable systems, such as bosonic modes of the electromagnetic field which are described by an infinite-dimensional Hilbert space.



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